PG–A-1465 **MMS–25X**

P.G. DEGREE EXAMINATION — JULY, 2022.

Mathematics

(CY 2020 & AY 2020 Batches Onwards)

Second Year

TOPOLOGY AND FUNCTIONAL ANALYSIS

Time : 3 hours Maximum marks : 70

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE of the following.

1. If X is a set and $\mathfrak{B}$ is a basis for a topology τ on X, then show that τ equal the collection of all union of elements of $\mathfrak{B}$.

2. State and prove uniform continuity theorem.

3. Show that every metrizable space is normal.

4. State (a) the uniform boundedness theorem and (b) the closed graph theorem.

5. If A_1 and A_2 are self-adjoint operators on H, then show that their product $A_1 A_2$ is self-adjoint if and only if $A_1 A_2 = A_2 A_1$.

6. State and prove the sequence lemma.

7. Show that the image of a compact space under a continuous map is compact.

8. If N is a normed linear space and x_0 is a non-zero vector in N, then show that there exists a functional f_0 in N^* such that $f_0(x_0) = \|x_0\|$.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE of the following.

9. (a) State and prove the uniform limit theorem.

   (b) Let (X, τ) be a topological space and $\mathcal{C}$ is a collection of open sets of X such that for each open set U of X and each $x \in U$, there exists $C \in \mathcal{C}$ such that $x \in C \subset U$. Show that $\mathcal{C}$ is a basis for the topology τ on X.

10. Let X be a simply ordered set having the least upper bounded property. In the order topology, show that each closed interval in X compact.

11. State and prove Urysohn metrization theorem.

12. State and prove the Hahn-Banach theorem.

13. Let H be a Hilbert space, and let f be an arbitrary functional in H^*. Show that there exists a unique vector y in H such that $f(x) = (x, y)$ for every x in H.

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PG–A–1466	**MMS–26X**

P.G. DEGREE EXAMINATION — JULY, 2022.

Mathematics

(CY 2020 & AY 2020 Batches onwards)

Second Year

OPERATIONS RESEARCH

Time : 3 hours Maximum marks : 70

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE of the following.

1. Use Simplex method to solve the following Linear programming problem.

   Maximize $z = 3x_1 + 2x_2$

   Subject to the constraints :

   $x_1 + x_2 \leq 4$, $x_1 - x_2 \leq 2$ and $x_1, x_2 \geq 0$.

2. Explain the shortest route algorithm.

3. Sketch the Branch and Bound method in integer programming.

4. Write a short note on pure birth model.

5. Solve the following non-linear programming problem

Minimize $f(x_1, x_2) = 3x_1^2 + x_2^2 + 2x_1x_2 + 6x_1 + 2x_2$

Subject to the constraint

$$2x_1 - x_2 = 4 .$$

6. Obtain the dual problem of the following linear programming problem.

Maximize $z = x_1 - x_2 + 3x_3$

Subject to the constraints

$$x_1 + x_2 + x_3 \le 10$$

$$2x_1 - x_3 \le 2$$

$$2x_1 - 2x_2 + 3x_3 \le 6$$

and $x_1, x_2, x_3 \ge 0.$

7. For the game with the following pay-off matrix, determine the optimum strategies and the value of the game

$$\begin{array}{cc} & \text{A} \\ \text{B} & \begin{pmatrix} 5 & 1 \\ 3 & 4 \end{pmatrix} \end{array}$$

8. Customers arrive at a box office window, being manned by single individual, according to a Poisson input process with a mean rate of 30 per hour. The time required to serve a customer has an exponential distribution with a mean 90 seconds. Find the average waiting time of a customer.

SECTION B — ($3 \times 15 = 45$ marks)

Answer any THREE of the following.

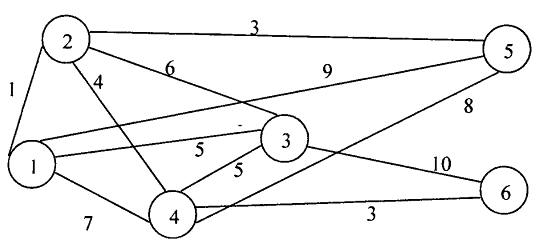
9. Solve the following linear programming problem by dual simplex method.

Minimize $z = 6x_1 + 7x_2 + 3x_3 + 5x_4$

Subject to the constraints

$$5x_1 + 6x_2 - 3x_3 + 4x_4 \geq 12$$
$$x_2 + 5x_3 - 6x_4 \geq 10$$
$$2x_1 + 5x_2 + x_3 + x_4 \geq 8$$
and $x_1, x_2, x_3, x_4 \geq 0$.

10. The Midwest T.V. Cable company is in the process of providing cable services to five new housing development areas. The figure below depicts the potential T.V. Linkages among the five areas. The cable miles are shown on each branch. Determine the most economical cable networking for Midwest Company.



Also determine the minimum spanning tree for the given network when

(a) Nodes 2 and 5 cannot be linked

(b) Nodes 5 and 6 are linked by a 2-mile cable.

11. Use cutting plane method to solve the following integer linear programming problem

Maximize $Z = 2x_1 + 2x_2$

Subject to the constraints

$$5x_1 + 3x_2 \leq 8$$

$$x_1 + 2x_2 \geq 4$$

$$x_1, x_2 \geq 0 \text{ and are integers.}$$

12. Find the optimum solution to the problem

Minimize $z = x_1^2 + x_2^2 + x_3^2$

Subject to

$$4x_1 + x_2^2 + 2x^3 - 14 = 0$$

13. Solving the following quadratic programming problem.

Maximize $Z = 2x_1 + x_2 - x_1^2$

Subject to the constraints

$$2x_1 + 3x_2 \leq 6$$

$$2x_1 + x_2 \leq 4$$

$$\text{and } x_1, x_2 \geq 0.$$

PG-A-1467	MMS–27X

P.G. DEGREE EXAMINATION — JULY, 2022.

Mathematics

(From CY – 2020 onwards)

Second Year

GRAPH THEORY AND ALGORITHMS

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE of the following.

1. Write Dijkstra's algorithm.

2. Show that a graph is 2-edge connected if and only if any two vertices are connected by at least two edge disjoint paths.

3. Prove that connected graph G is Eulerian if and only if each vertex of G has even degree.

4. Show that a non-empty graph G is 2-colourable if and only if G is bipartite.

5. Show that every planar graph is 6-vertex colourable.

6. State and prove pigeonhole principle.

7. Show that closure of a graph is well defined.

8. In a critical graph, show that no vertex cut is a clique.

PART B — $(3 \times 15 = 45$ marks)

Answer any THREE of the following.

9. (a) Show that a graph is bipartite if and only if it contains no odd cycle.

   (b) Show that every connected graph has at least two vertices that are not cut vertices.

10. For any Graph G, show that $\kappa(G) \le \lambda(G) \le \delta(G)$.

11. Write Fleury's algorithm and show that Fleury's algorithm produces a closed Euler trail in G if G is an Euler graph.

12. State and prove Vizing's theorem.

13. State and prove five-colour theorem.

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PG-A-1468	**MMS-28X**

P.G. DEGREE EXAMINATION – JULY 2022.

Mathematics

(From CY – 2020 Onwards)

Second Year

DIFFERENTIAL EQUATIONS

Time : 3 hours Maximum marks : 70

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Find the particular solution of $y''+y=\operatorname{cosec} x$.

2. Prove that $P_n(-x)=(-1)^n P_n(x)$ and hence $P_n(-1)=(-1)^n$.

3. Solve the initial value problem $y''-2y'+y=0$ $y(0)=0$ and $y'(0)=1$ on the bounded interval [0, a] where a is a perimeter.

4. If u is complementary function and if z_1 is a particular integral of the partial differential equation $F(D.D') = f(x,y)$ then prove that $u + z_1$ is the general solution of $F(D.D') = f(x,y)$.

5. Show that the necessary and sufficient condition for the existence of the solution of the interior Neumann's problem is that the integral of f over the boundary S should vanish .

6. Solve $3y' + y = 2e^{-x}$.

7. Find the fundamental matrix of the equation
$$y' = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 3 \\ 0 & 1 & 0 \end{bmatrix} y.$$

8. Classify the equation $u_{xx} + u_{yy} = u_{xy}$.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE questions.

9. Find the solution of the initial value problem $y'' - 2y' + y = 2x$, $y(0) = 6$ and $y'(0) = 2$.

10. Derive the Bessel functions of zero order of first kind.

11. State and prove existence and uniqueness theorem for the matrix $A(x)$ is continuous, the system of equation $y' = A(x) \quad y(x_0) = y_0 (x, x_0 \in 1)$.

12. Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ into its canonical form.

13. State and prove Kelvin's inversion theorem.

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